Homework 1 Solutions

Ross Summer Connection 2021

Due: Sunday August 1st @ 11:59pm

Question 1: Pat and Kris are roommates. They spend time making pizza and brewing root beer. Pat takes 4 hours to brew a gallon of root beer and 2 hours to make a pizza. Kris takes 6 hours to brew a gallon of root beer and 4 hours to make a pizza.

a) Assume there are 12 hours available for production. Construct a table representing the maximum amount of root beer and pizza that each person could produce with their time.

	Pat	Kris
Pizza	6	3
Root Beer	3	2

b) Assuming constant opportunity cost, construct a table describing how much of the other good each person would have to give up to get another unit of one good.

	Pat	Kris
1 Pizza	1/2	2/3
1 Root Beer	2	3/2

c) Who has the absolute advantage in making pizza? In root beer? (Note: absolute advantage means who can make most of each thing given total time, while comparative advantage means who has the give up the least of the other thing to make another unit of the first!)

From part a, Pat has the absolute advantage in making pizza and root beer!

d) Who has the comparative advantage in making pizza? In root beer?

From part b, Pat has a comparative advantage in producing pizza while Kris has a comparative advantage in producing root beer.

e) If Pat and Kris trade foods with each other, who will trade away pizza in exchange for root beer? (Hint: economic agents with a comparative advantage in the production of some good will specialize in that good and sell it in exchange for the other good.)

Pat will trade pizza to Kris in exchange for root beer.

f) The price of pizza can be expressed in terms of gallons of root beer. What is the highest price at which pizza can be traded that makes both people better off? What is the lowest? (Hint: think about which prices must hold for Pat and Kris to be willing to trade.)

Since Kris is receiving pizza from Pat, she will not be willing to pay more than her own (opportunity) cost of producing pizza, which is 2/3. Thus, the maximum price of pizza in terms of root beer is 2/3. Similarly, the minimum price of pizza is Pat's opportunity cost, which is 1/2. Any other prices will make either Pat or Kris unwilling to trade.

Question 2: What is the opportunity cost of a banana in terms of a coconut if a banana costs \$2 and a coconut costs \$5? What is the opportunity cost of coconuts in terms of bananas? (Assume you can buy and sell fractions of a fruit!)

First, consider the opportunity cost of coconuts in terms of bananas. It costs \$2 to buy a banana and \$5 to buy a coconut. I want to know how many bananas I would have to give up to buy a coconut. This question is the same as, how many bananas could I buy for \$5? The answer is 2.5 bananas. Thus, $\boldsymbol{OC_{CB} = 2.5} = \frac{5}{2}$, which also happens to be the price ratio! Note that this will generally be true; when prices are given in terms of money, price ratios give oportunity costs!

Question 3: Roger's Carpet Cleaning Business faces the following total costs and total benefits associated with cleaning carpets. Answer the following questions using the information given.

a) In two new columns, find marginal cost and marginal benefit. What is the optimal quantity?

Q	TC	TB	MC	MB
0	0	0	–	–
1	20	60	20	60
2	42	110	22	50
3	68	150	26	40
4	100	180	32	30
5	140	200	40	20

The optimal quantiy is $\boldsymbol{Q^* = 3}$, which is the last place $\boldsymbol{MB(Q) \geq MC(Q)}$.

b) In a new column, find total profits. What is the optimal quantity?

Q	TC	TB	Profits
0	0	0	0
1	20	60	40
2	42	110	68
3	68	150	82
4	100	180	80
5	140	200	60

The optimal quantity is again $\boldsymbol{Q^* = 3}$, which is where profits are maximized.

c) Plot marginal cost and marginal benefit on a graph. What is the optimal quantity?

The graph will have an upward sloping marginal cost curve and a downward sloping marginal benefit curve, which intersect at a quantity of $\boldsymbol{Q^* = 3}$ units, the optimum.

Question 4: April is a promising young scientist who needs to decide how much time to spend analyzing data with her microscope. Her *total benefit* from spending t hours on the scope is given by $TB(t) = At$ while her *marginal cost* is $MC(t) = t^2 + B$, where $A > B > 0$ are constants.

a) Find the optimal time t^* spent on the microscope, and show your answer graphically.

Since we have continuous benefit and cost functions, we can use the rule that the optimal time $\boldsymbol{t^*}$ must occur at the intersection of the marginal cost curve $\boldsymbol{MC(t)}$ and the marginal benefit curve $\boldsymbol{MB(t)}$. We must derive the marginal benefit curve from the total benefit function, and we can do so assuming that time increases by a single unit from $\boldsymbol{t}$ to $\boldsymbol{t+1}$.

$$\boldsymbol{MB(t) = \frac{\Delta TB}{\Delta t} = \frac{A \cdot (t+1) - A \cdot t}{(t+1) - t} = A}$$

Intuitively, the extra benefit from increasing time by one unit is just the constant $\boldsymbol{A}$ because the total benefit function is linear. We can then find the optimal time by setting

$$\boldsymbol{\begin{aligned} MB(t) &= MC(t) \\ A &= t^2 + B \\ t^* &= \sqrt{A - B} \end{aligned}}$$

which is guaranteed to be a real number since we have assumed that $\boldsymbol{A > B > 0}$.

a) What happens to the optimal solution when A gets bigger? Describe why this mathematical result is equivalent to an increase in April's productivity.

The optimal time on the scope (aka the solution) increases as $\boldsymbol{A}$ gets bigger, which makes sense since a higher value of $\boldsymbol{A}$ is equivalent to an increase in the marginal productivity of a unit of April's time. We know this because her marginal benefit function is $\boldsymbol{A}$!

Question 5: Suppose that there are two countries, with 70 people in the poor (P) country and 30 people in the rich (R) country. Labor markets are competitive in both countries, so workers are paid according to their marginal products. Thus, wages are given by

$$\begin{aligned} w_P &= MP_P(L) \coloneqq 7 - 0.07L \\ w_R &= MP_R(L) \coloneqq 10 - 0.03L \end{aligned}$$

a) Explain what the following statement means and give an example: the marginal product of labor in production is (often) diminishing, or getting smaller, with additional workers.

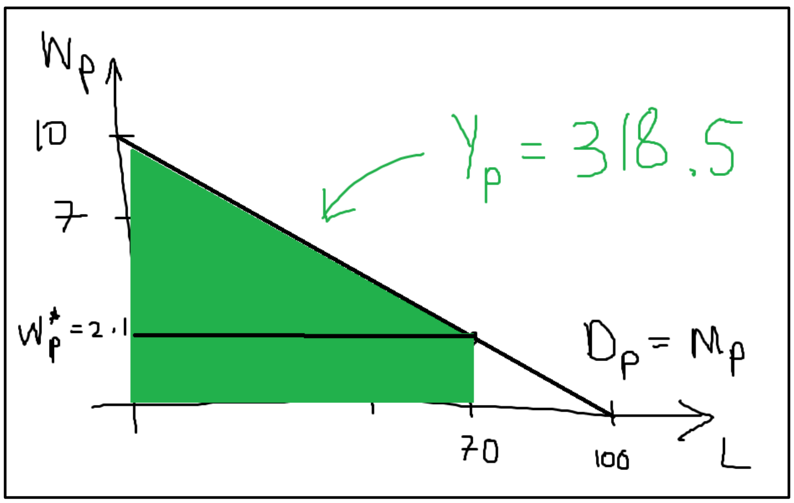
Workers generally contribute to production, but successive workers are increasingly less productive. This is because we assume that we are studying the short run, where the capital stock at the firm (aka the number of machines) is fixed. Thus, the n-th worker will have a lower marginal product than the one before them, because there will be fewer machines (per worker!) once they join the team; this usually holds for all n. Examples of this exist everywhere: restaurants, barber shops, manufacturing, and even teaching!

b) How do you know that both of these countries satisfy diminishing marginal products?

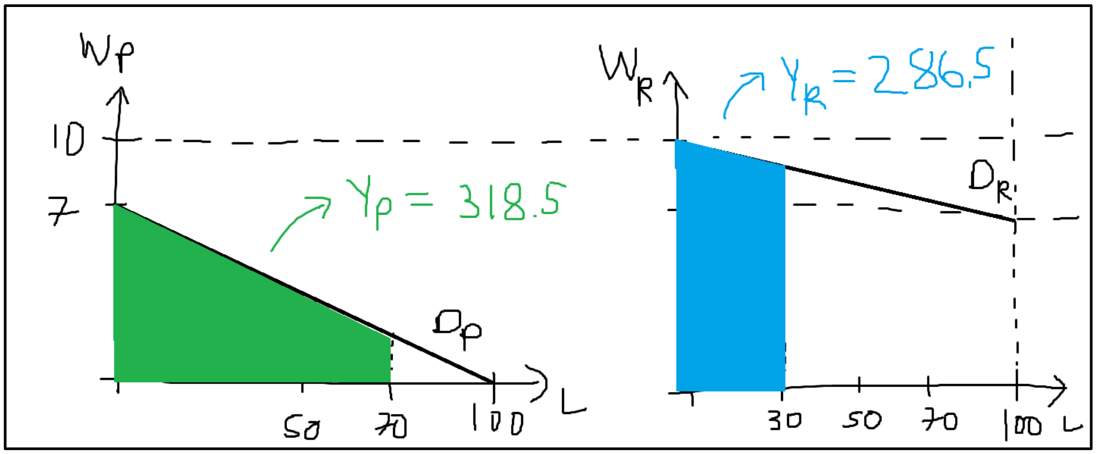
The marginal product functions are decreasing in labor L. (That is, they slope downwards!)

c) In a graph with w_P on the vertical axis and L on the horizontal axis, plot the poor country's labor demand curve. Assuming full employment, find total output. (Hint: Trapezoids!)

Total output is $\boldsymbol{Y_P = 318.5}$.



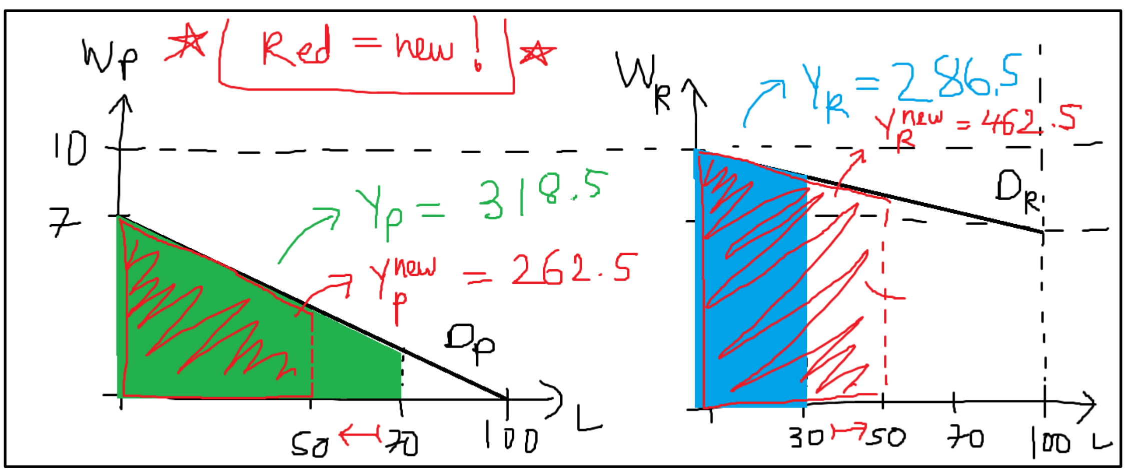
d) Assuming full employment, find total world out. (Hint: repeat c for the rich country.)



We now have $\boldsymbol{Y = Y_P + Y_R = 318.5 + 286.5 = 605}$. (Note that the poor country actually produces more than the rich country in this question! While I didn't intend this when I set parameter values, it illustrates an important point; GDP per capita is often a more useful measure of well-being, in the data, than overall output.)

e) Now assume rich country citizens have read Clemens (2011) and reflected on the state of the currently segregated world. They decide to allow 20 poor country citizens to migrate. Repeat steps c and d, again assuming full employment.

Repeat solutions above! You'll get $\boldsymbol{Y^{new} = 725}$.



f) Compute the percent increase in output. (And sing it from the rooftops!)

The model-implied increase in world output is $\boldsymbol{\Delta_M = \frac{Y^{new} - Y}{Y} \approx 19.8\%}$.

g) Replicate Figure 1 in Clemens (2011) to the best of your ability.

See Figure 1 of Clemens (2011), with appropriate labels! The gray area is equal to about 20% of total world output, in this model with two countries and only labor production.

h) In Haxhiu (2020), I show that emigration can actually *improve* the productivity of non-migrants back home through increased education financed by remittances. Suppose that after those 20 poor citizens leave, the labor demand curve changes to

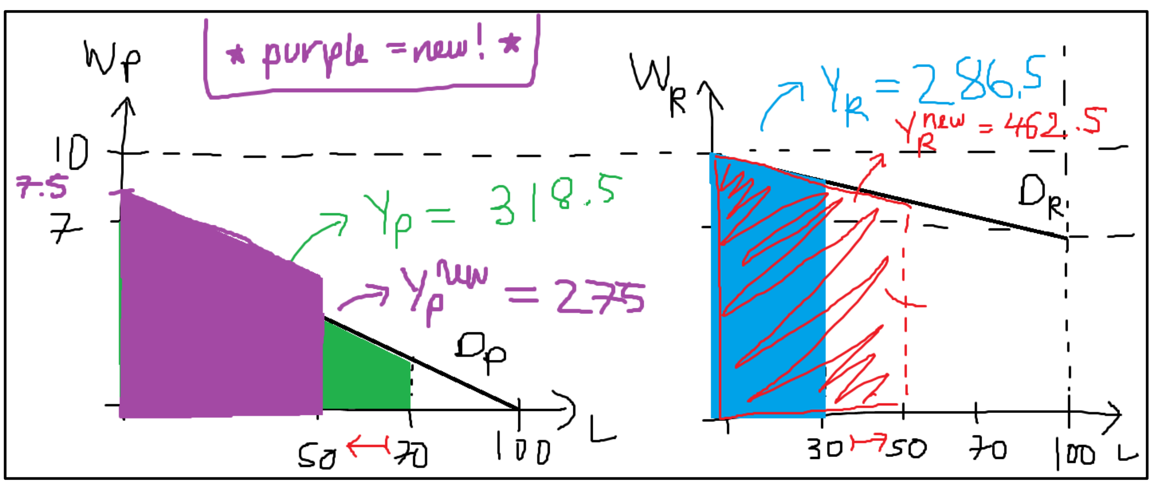
$$w_P^{NEW} = 7.5 - 0.07L$$

Explain why this mathematical result is equivalent to a positive externality from migration!

As a result of migration, the productivity (aka wage, which is the same thing if we assume perfect competition) of non-migrants increases (higher slope). It is an externality because the behavior of one group (migrants) affects the "price" faced by another (non-migrants, where price refers to the wage), and it is positive because the non-migrants benefit!

i) Compute the percent increase in output, considering this positive externality, and compare it to your answer in part f.

We now have $\Delta_M^{EXT} = \frac{Y^{new} - Y}{Y} \approx \mathbf{21.9\%}$.



j) Suppose the migrants face discrimination in the rich country, and only receive half of the posted wage. How would you represent this change mathematically in the model, and how would it change our conclusions?

Their actual marginal product (or wage) curve lies below that described above, which now only applied to natives, for whom we assume discrimination does to apply. It changes some conclusions, but not the substance of the message: free mobility is still overwhelmingly beneficial from a welfare standpoint. Although the migrants face a wage penalty, everybody still wins (on net, with distributional concerns notwithstanding) since those migrants for whom the discrimination penalty is too sever do not migrate (the very nature of mobility "freedom"). Many will still likely find it beneficial, however, and they still contribute to production, so world output will still increase dramatically (although migrants don't receive the marginal product of their labor, which seems unlegit.)

Question 6 (EXTRA CREDIT): Recall that in LEC1 we introduced marginal analysis by studying the optimal schooling decision given total benefit and cost functions. As some of you have already noted, it is unrealistic to think that we know all the functions: TB, TC, MB, MC. In this problem, we will see how causal inference can be used to learn about certain aspects of marginal benefits!

a) Let $Y_i \geq 0$ denote hourly earnings of individual i (aka outcome). Let $D_i \in \{0,1\}$ denote whether enrolled in college. We can write earnings as a function of enrollment status: $Y_i(1)$ and $Y_i(0)$, which are called potential outcomes. Write down the average treatment effect (ATE) in terms of potential outcomes and explain what it measures.

   We wish to estimate the value of the average treatment effect (ATE)

$$\boldsymbol{ATE \coloneqq E[\tau_i] = E[Y_i(1) - Y_i(0)]}$$

   Which gives the average effect on earnings of getting a college degree for everyone.

b) If you know whether someone is enrolled in college, write down actual hourly earnings as a function of potential outcomes and enrollment status.

   Note that we can write observed earnings in terms of potential earnings if we know whether the person went to college:

$$\boldsymbol{Y_i = D_i Y_i(1) + (1 - D_i) Y_i(0)}$$

   so that $\boldsymbol{Y_i = Y_i(1)}$ if $\boldsymbol{D_i = 1}$ and vice versa.

c) Write down actual earnings from part b in terms of the individual treatment effect τ_i.

With some algebra we have

$$\begin{aligned} \boldsymbol{Y_i} &= \boldsymbol{D_i Y_i(1) + Y_i(0) - D_i Y_i(0)} \\ &= \boldsymbol{Y_i(0) + D_i[Y_i(1) - Y_i(0)]} \\ &= \boldsymbol{Y_i(0) + D_i \cdot \tau_i} \end{aligned}$$

d) Suppose that even without the college education, those who eventually went on to enroll in college would have higher earnings. Describe what this means for the selection bias term $SB \coloneqq E[Y_i(0)|D_i = 1] - E[Y_i(0)|D_i = 0]$.

In this scenario, we would have SB > 0 since $\boldsymbol{E[Y_i(0)|D_i = 1] > E[Y_i(0)|D_i = 0]}$. In other words, we have positive selection bias in this example.

e) Would a simple comparison of those enrolled versus not enrolled in college like

$$E[Y_i|D_i = 1] - E[Y_i|D_i = 0]$$

be a biased estimate of the ATE (under scenario in part c). If so, is it biased up or down?

Yes, it would be biased since even without the college education, those who eventually went on to enroll in college would have higher earnings. We see that this leads to an upwards biased estimate of the ATE since

$$\boldsymbol{E[Y_i|D_i = 1] - E[Y_i|D_i = 0] = E[Y_i(1)|D_i = 1] - E[Y_i(0)|D_i = 0]}$$

$$\begin{aligned} &= \boldsymbol{E[Y_i(1)|D_i = 1] - E[Y_i(0)|D_i = 0]} \\ &\quad \boldsymbol{+ E[Y_i(0)|D_i = 1] - E[Y_i(0)|D_i = 1]} \end{aligned}$$

$$\begin{aligned} &= \boldsymbol{E[Y_i(1) - Y_i(0)|D_i = 1] + E[Y_i(0)|D_i = 1] - E[Y_i(0)|D_i = 0]} \\ &> \boldsymbol{ATE} \end{aligned}$$

since SB > 0.