ECON 402 Discussion: Week 1 (lecture)

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Announcements

- Elird Haxhiu
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- Lorch M101

- Lectures: Fridays at 11am, AH G127
- Problems: Fridays at 2pm, AH G127
  - Both recorded and posted to Canvas
  - Live attendance to both is encouraged if possible!

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- Topics today
  1. Growth rates
  2. Production functions
  3. Growth accounting
  4. Solow model environment

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- We study different definitions of GE, and the implied restrictions on the evolution of macroeconomic variables like output Y_t, wages w_t, investment spending I_t, or the aggregate saving rate $s \in (0, 1)$
  - A theory is simply an implied restriction on the data... aka prediction!
  - Empirics: use econometrics to test predictions, given id assump.
  - We'll do the first part, and we'll do it "rigorously" but gently!

Growth Rates

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- Note: In continuous time, let $g_x := \frac{\dot{x}_t}{x_t}$ where $\dot{x}_t = \frac{d}{dt} x_t$.

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- Example: Prove that $g_x \approx \ln x_{t+1} - \ln x_t$ if g_x if sufficiently small.

- Proof: In discrete time, we have

$$\begin{aligned} \ln x_{t+1} - \ln x_t &= \ln\left(\frac{x_{t+1}}{x_t}\right) = \ln\left(\frac{(1+g_x)x_t}{x_t}\right) \\ &= \ln(1+g_x) \approx g_x \end{aligned}$$

by Taylor's approximation theorem...throwback! In continuous time, we have $\frac{d}{dt}\ln x_t = \frac{1}{x_t}\frac{d}{dt}x_t = \frac{\dot{x}_t}{x_t} = g_x$ (no longer an approximation).

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- Solution: In discrete time, we have

$$g_Y = \frac{Y_1 - Y_0}{Y_0} = \frac{[200 + 10] - [0 + 10]}{0 + 10} = 20$$

while in continuous time

$$g_Y = \frac{\dot{Y}_t}{Y_t} = \frac{200}{200t + 10} = \frac{200}{200 \cdot 0 + 10} = 20$$

- Note: these are only the same because Y_t is linear in time!

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- Solution: We begin with the definition of y_t, take logs, and then differentiate with respect to time

$$\begin{aligned} y_t &= Ax_t^B \\ \ln y_t &= \ln\left(Ax_t^B\right) = \ln A + B \ln x_t \end{aligned}$$

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\frac{d}{dt}\left[\ln y_t\right] &= \frac{d}{dt}\left[\ln A + B \ln x_t\right] \\
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- Note: This illustrates the general principal of log differentiating a variable with respect to time to compute its growth rate.

Production Functions

- Definition: Let Y_t denote output, K_t denote the capital input, and L_t denote the labor input. Then $F : \mathbb{R}^3_+ \to \mathbb{R}$ is a production function if $Y_t := F(K_t, L_t, A)$, where $A > 0$ is constant.

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  3. [MPs diminishing] $F_{KK} := \frac{\partial^2}{\partial K^2} F < 0$ and $F_{LL} := \frac{\partial^2}{\partial L^2} F < 0$,
  4. [CRS] For all $\lambda > 0$, we have $F(\lambda K_t, \lambda L_t, A) = \lambda F(K_t, L_t)$.

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- Note: Cobb-Douglas production functions also satisfy the other three neoclassical assumptions. You should be able to prove them!

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Solow Model

Definition: The building blocks of the Solow growth model are

- Production: $Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$ with $\alpha \in (0, 1)$.
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- Per capita quantities: $k_t = \frac{K_t}{L_t}$, $y_t = \frac{Y_t}{L_t}$, and $c_t = \frac{C_t}{L_t}$.
- Input prices (under perfect competition): $R_t = F_K$ and $w_t = F_L$.