ECON 402 Discussion: Week 1 (problems)

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Announcements

- Elird Haxhiu
- haxhiu@umich.edu
- Lorch M101

- Lectures: Fridays at 11am, AH G127
- Problems: Fridays at 2pm, AH G127
  - Both recorded and posted to Canvas
  - Live attendance to both is encouraged if possible!

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- Homework: posted weekly, both easy and hard, for extra credit
- How to get the most out of this?

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- Topics today
  1. Solow model environment
  2. Golden rule savings rate
  3. Growth problems

Solow Model

Definition: The building blocks of the Solow growth model are

- Production: $Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$ with $\alpha \in (0, 1)$.
- Accounting: $Y_t = C_t + I_t$ with $G_t = NX_t = 0$ for now.

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- Laws of motion for inputs:
  - $\dot{K}_t = I_t - \delta K_t$ where $\delta \in (0,1)$,
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- Per capita quantities: $k_t = \frac{K_t}{L_t}$, $y_t = \frac{Y_t}{L_t}$, and $c_t = \frac{C_t}{L_t}$.
- Input prices (under perfect competition): $R_t = F_K$ and $w_t = F_L$.

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- Step 1: Find the law of motion for the capital-labor ratio k_t.

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&= s \frac{A \cdot K_t^{\alpha} L_t^{1-\alpha}}{L_t} - (\delta + n) k_t
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- Step 3: Find consumption per capita in steady state c_*.

$$c_t \;:=\; \frac{C_t}{L_t} = \frac{(1-s)Y_t}{L_t} = (1-s)k_t^\alpha$$

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- Step 4: Solve the first-order condition (FOC) $\frac{\partial}{\partial s} c_*(s) = 0$ for s.

$$\frac{\partial}{\partial s}\left[(1-s)\left(\frac{sA}{\delta+n}\right)^{\frac{\alpha}{1-\alpha}}\right] = 0$$

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s_* &= \alpha
\end{aligned}
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Problem 1

- Consider a Solow type economy. Assume that initially the saving rate is below the golden rule of saving s_*, and that the economy is in steady state. Imagine that the government implements a policy to increase the saving rate all the way to s_*. What is the impact of such a policy on the level of consumption in the long run?
  - A Same
  - B Increase
  - C Decrease
  - D Indeterminate

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Problem 3

- Imagine that a Solow type economy experiences a one time improvement in total factor productivity A_t. What is the impact of such an impulse on the level of consumption in the long run?
  - A Unchanged
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Problem 4

- Assume that the saving rate in the US is only 10% and in China it is equal to 40%, but TFP in the US is 40 and in China it is only equal to 10. In which of the two countries will the *level of capital per person* be higher in the long run?
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