ECON 402 Discussion: Week 1

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Announcements

- Elird Haxhiu
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- Lorch Hall M101 (Mezzanine)

- Discussion: Thursdays at 12pm, Mason Hall 1469
- Office hours: Fridays at 3pm - 5pm, Lorch Hall M101
- How to get the most out of this?

- Topics today
  1. Growth rates
  2. Production functions
  3. Profit maximization

Economics: study allocation of scarce resources

- Better: study human behavior as decisions made by individuals who maximize objective functions, constrained by budget sets and subject to equilibrium conditions... sometimes
  - Why sometimes? Partial versus General Equilibrium (GE)
  - And hence, *one* distinction between Micro and Macroeconomics

- We study different definitions of GE, and the implied restrictions on the evolution of macroeconomic variables like output Y_t, wages w_t, investment spending I_t, or the aggregate saving rate $s \in (0, 1)$

- A theory is simply an implied restriction on the data... aka prediction
- Empirics: use econometrics to test predictions, given id assump
- We'll do the first part!

Growth Rates

- Let x_t be some economic variable measured in discrete time $t \in \mathbb{N}$. The net growth rate g_x and gross growth rate G_x are defined as

$$
\begin{aligned}
g_x &:= \frac{x_{t+1} - x_t}{x_t} \\
G_x &:= \frac{x_{t+1}}{x_t} = 1 + g_x
\end{aligned}
$$

- Example: Let Y_t denote GDP per capita in the United States. Then we usually have $g_Y = 0.03$ and $G_Y = 1.03$. (Careful with decimal versus percent notation!)

- Note: In continuous time, let $g_x := \frac{\dot{x}_t}{x_t}$ where $\dot{x}_t = \frac{d}{dt} x_t$.

Growth Rates

- If g_x is constant over time, then we can write

$$\begin{aligned} x_{t+1} &= (1+g_x)x_t = G_x x_t \\ \Rightarrow x_{t+n} &= (1+g_x)^n x_t = G_x^n x_t \end{aligned}$$

- Example: Prove that $g_x \approx \ln x_{t+1} - \ln x_t$ if g_x if sufficiently small.
- Proof: In discrete time, we have

$$\ln x_{t+1} - \ln x_t = \ln\left(\frac{x_{t+1}}{x_t}\right) = \ln\left(\frac{(1+g_x)x_t}{x_t}\right) = \ln(1+g_x) \approx g_x$$

by Taylor's approximation theorem. In continuous time, no longer an approximation since

$$\frac{d}{dt}\ln x_t = \frac{1}{x_t}\frac{d}{dt}x_t = \frac{\dot{x}_t}{x_t} = g_x$$

Growth Rates

- Example: If x_t grows at constant rate g_x, find the growth rate of the variable $y_t := Ax_t^B$ where $A, B \in \mathbb{R}$.

- Solution: Given the definition of y_t, take logs and differentiate with respect to time

$$
\begin{aligned}
y_t &= Ax_t^B \\
\ln y_t &= \ln\left(Ax_t^B\right) = \ln A + B \ln x_t \\
\frac{d}{dt}\left[\ln y_t\right] &= \frac{d}{dt}\left[\ln A + B \ln x_t\right] \\
\frac{\dot{y}_t}{y_t} &= 0 + B\frac{\dot{x}_t}{x_t} \\
g_y &= Bg_x
\end{aligned}
$$

- General principal: log differentiate a variable with respect to time to find its growth rate!

- Let Y_t denote output, K_t denote the capital input, and L_t denote the labor input. Then $F : \mathbb{R}^3_+ \to \mathbb{R}$ is a production function if $Y_t := F(K_t, L_t, A)$, where $A > 0$ is constant.

- Neoclassical assumptions on production
    1. [Continuity] F is continuous and (twice) differentiable
    2. [Marginal Products > 0] $F_K := \frac{\partial}{\partial K} F > 0$ and $F_L := \frac{\partial}{\partial L} F > 0$
    3. [MPs diminishing] $F_{KK} := \frac{\partial^2}{\partial K^2} F < 0$ and $F_{LL} := \frac{\partial^2}{\partial L^2} F < 0$
    4. [Constant Returns to Scale] For all $\lambda > 0$, we have $F(\lambda K_t, \lambda L_t, A) = \lambda F(K_t, L_t)$

Cobb-Douglas Production Functions

- $Y_t = AK_t^{\alpha} L_t^{\beta}$ for constant $\alpha > 0$ and $\beta > 0$... it's very famous!
- Example: For which values of $\alpha > 0$ and $\beta > 0$ does the Cobb Douglas production function exhibit constant returns to scale (CRS)?

- Solution: We start by scaling all factors by a constant

$$\begin{aligned} F(\lambda K_t, \lambda L_t, A) &= A(\lambda K_t)^{\alpha}(\lambda L_t)^{\beta} \\ &= \lambda^{\alpha+\beta} A K_t^{\alpha} L_t^{\beta} \\ &= \lambda F(K_t, L_t, A) \end{aligned}$$

  if and only if $\alpha + \beta = 1$

- Normalizing $\alpha + \beta = 1$ is a standard assumption precisely because we want CRS!
- Cobb-Douglas production functions also satisfy the other three neoclassical assumptions. You should be able to prove them!

Profit Maximization Example

Suppose there exists a representative firm in the economy with Cobb-Douglas production function $Y_t = K_t^\alpha L_t^{1-\alpha}$ for $\alpha \in (0,1)$ and output price P normalized to 1.

a) Write out the firm's long-run profit function.

$$\begin{aligned} \pi(K_t, L_t) &= P \cdot F(K_t, L_t) - R_t \cdot K_t - w_t \cdot L_t \\ &= K_t^\alpha L_t^{1-\alpha} - R_t K_t - w_t L_t \end{aligned}$$

b) Write out the firm's short-run profit function assuming capital is fixed at $\overline{K}$.

$$\begin{aligned} \pi(\overline{K}, L_t) &= P \cdot F(\overline{K}, L_t) - R_t \cdot \overline{K} - w_t \cdot L_t \\ &= \overline{K}^\alpha L_t^{1-\alpha} - R_t \overline{K} - w_t L_t \end{aligned}$$

c) Solve the firm's profit maximization problem in the short-run, where capital is fixed at some level $\overline{K}$. What is the wage rate? What is the labor demand (LD) curve? What shifts LD exogenously?

d) Find the long-run optimal capital-labor ratio $k_t := \frac{K_t}{L_t}$ by solving the firm's profit maximization problem. (Note: why can't we solve for unique values or capital K_t and labor L_t that are optimal in the long-run in this case?)