ECON 402 Discussion: Week 2

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Announcements

- *NEW* Office hours: Mondays at 3pm - 5pm, Lorch Hall M101
- Homework 1 posted and due Wednesday January 25th

- Topics today
  1. Production functions
  2. Profit maximization
  3. Markets for labor, capital, and loan-able funds

Neoclassical Production Functions $Y = F(K, L)$

1. [Continuity] F is continuous and twice differentiable
2. [Marginal Products > 0] $F_K := \frac{\partial}{\partial K} F > 0$ and $F_L := \frac{\partial}{\partial L} F > 0$
3. [MPs diminishing] $F_{KK} := \frac{\partial^2}{\partial K^2} F < 0$ and $F_{LL} := \frac{\partial^2}{\partial L^2} F < 0$
4. [Constant Returns to Scale] For all $\lambda > 0$, we have $F(\lambda \cdot K, \lambda \cdot L) = \lambda \cdot F(K, L)$

Profit Maximization Example

Suppose there exists a representative firm in the economy with Cobb-Douglas production function $Y_t = K_t^\alpha L_t^{1-\alpha}$ for $\alpha \in (0,1)$ and output price P normalized to 1.

a) Write out the firm's long-run profit function.

$$\begin{aligned}\pi(K_t, L_t) &= P \cdot F(K_t, L_t) - R_t \cdot K_t - w_t \cdot L_t \\ &= K_t^\alpha L_t^{1-\alpha} - R_t K_t - w_t L_t\end{aligned}$$

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b) Write out the firm's short-run profit function assuming capital is fixed at $\overline{K}$.

$$\begin{aligned} \pi(\overline{K}, L_t) &= P \cdot F(\overline{K}, L_t) - R_t \cdot \overline{K} - w_t \cdot L_t \\ &= \overline{K}^{\alpha} L_t^{1-\alpha} - R_t \overline{K} - w_t L_t \end{aligned}$$

c) Solve profit maximization in the short-run, where capital is fixed at some level $\overline{K}$. What is the wage rate? What is the labor demand (LD) curve? What shifts LD exogenously?

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The firm's problem is $\max_{L_t} \pi(\overline{K}, L_t)$. Since the production function F is Neoclassical, the solution is given by the first-order condition (FOC)

$$FOC(L) : \frac{\partial}{\partial L_t}\pi(\overline{K}, L_t) = (1-\alpha)\overline{K}^{\alpha} L_t^{-\alpha} - w_t \overset{!}{=} 0$$

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Profit Maximization Example

d) Find the long-run optimal capital-labor ratio $k_t := \frac{K_t}{L_t}$ by solving the firm's profit maximization problem. (Note: why can't we solve for unique values or capital K_t and labor L_t that are optimal in the long-run in this case?)

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$$FOC(K) \quad : \quad \frac{\partial}{\partial K_t}\pi(K_t, L_t) = \alpha K_t^{\alpha-1}L_t^{1-\alpha} - R_t \overset{!}{=} 0$$

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We take the ratio FOC(K)/FOC(L) which gives

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\frac{\alpha}{1-\alpha} K_t^{-1} L_t &= \frac{R_t}{w_t} \\
k_t &= \frac{\alpha}{(1-\alpha)} \frac{w_t}{R_t}
\end{aligned}
$$

Some important things to know...

1. Graph capital and labor markets under perfect competition (and fixed supply)
2. Relate wages and rental rates on capital to production function derivatives (supply side)
3. Divide up total output between factors under constant returns to scale
4. Specify consumption and investment as functions of real interest rate (demand side)
5. Connect goods market clearing ($Y = C + I + G$) and investment savings ($I = S$) identities

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\begin{aligned}
P \cdot \frac{\partial F}{\partial K} &= R \quad \Leftrightarrow \quad \text{MPK} = \frac{R}{P} \\
P \cdot \frac{\partial F}{\partial L} &= w \quad \Leftrightarrow \quad \text{MPL} = \frac{w}{P}
\end{aligned}
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$$\begin{aligned} F(K, L) &= \frac{\partial F}{\partial K} \cdot K + \frac{\partial F}{\partial L} \cdot L \\ Y &= \text{MPK} \cdot K + \text{MPL} \cdot L \\ Y &= \frac{R}{P} \cdot K + \frac{w}{P} \cdot L \end{aligned}$$

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$$\begin{aligned} C_t &= C(Y_t - T_t, r) \quad \text{where } \frac{\partial C}{\partial r} < 0 \\ I_t &= I(r) \quad \text{where } \frac{\partial I}{\partial r} < 0 \quad \text{follows} \quad \text{MPK} - \delta \cdot \frac{P_K}{P} = r \cdot \frac{P_K}{P} \end{aligned}$$

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$$\begin{aligned} Y &= C + I + G \\ Y - C - G &= I \\ S &= I \end{aligned}$$