ECON 402 Discussion: Week 3 (lecture + probs)

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Announcements

- This week only: no 2pm problem discussion, both recorded at 11am
- Quiz 1 last week: Solow, growth, and consumption models
- Quiz 2 next week: debts, deficits, and IS-LM/AS-AD models

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- Topics today
  1. Introduction
  2. Labor supply
  4. Quiz 1 discussion

Introduction

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- To spend, the government must (eventually) raise money by taxing its citizens. Different types of taxes can “distort” economic behavior in different ways, and we will consider some today.

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- To spend, the government must (eventually) raise money by taxing its citizens. Different types of taxes can "distort" economic behavior in different ways, and we will consider some today.

- Why eventually? Spending through debt is another option!
- Main prediction of more exotic models (overlapping generations...) is that the equilibrium could be "dynamically inefficient" so there's scope for government debt to help...

Labor Supply

- Example: Suppose an agent has utility function

$$u(C_t, L_t) = C_t - \frac{1}{2}L_t^2$$

where C_t is consumption and L_t is hours worked, or labor supply. Recall that there are $T_t := L_t + \ell_t$ hours in the day, so ℓ_t is leisure.

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- Part a: If the government imposes a tax $\tau \in (0, 1)$ on hours worked, find optimal labor supply.
- Solution: The agent solves

$$\max_{C_t, L_t} \left\{ C_t - \frac{1}{2}L_t^2 \right\} \quad \text{s.t.} \quad C_t = (1-\tau)\ w_t L_t$$

$$\Leftrightarrow \quad \max_{L_t} \left\{ (1-\tau)\ w_t L_t - \frac{1}{2}L_t^2 \right\}$$

Labor Supply

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- Part b, the (in?)famous Laffer Curve: Find (and plot) government tax revenue as a function of τ.
- Solution: Government revenues are

$$\begin{aligned} G_t(\tau) &:= \tau \cdot w_t L_t^*(w_t,\tau) \\ &= \tau(1-\tau)w_t^2 \end{aligned}$$

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$$\begin{aligned} G_t'(\tau) &\overset{!}{=} 0 \\ w_t^2(1 - 2\tau) &= 0 \end{aligned}$$

$$\begin{aligned} \Rightarrow \tau^* &= \frac{1}{2} \\ G_t(\tau^*) &= \frac{1}{2}\left(1 - \frac{1}{2}\right) w_t^2 = \frac{1}{4} w_t^2 \end{aligned}$$

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- Solution:

$$
\begin{aligned}
U_t^* &:= u(C_t^*, L_t^*) \\
&= C_t^* - \frac{1}{2}(L_t^*)^2 \\
&= (1-\tau)w_t L_t^* - \frac{1}{2}(L_t^*)^2 \\
&= (1-\tau)w_t \cdot (1-\tau)w_t - \frac{1}{2}((1-\tau)w_t)^2 \\
&= \frac{1}{2}(1-\tau)^2 w_t^2
\end{aligned}
$$

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&= \frac{1}{2} w_t^2 \cdot (1 - 2\tau + \tau^2) \\
&= \frac{1}{2} w_t^2 \cdot (1 - 2\tau + 2\tau^2 - \tau^2) \\
&= \frac{1}{2} w_t^2 \cdot (1 - 2\tau(1-\tau) - \tau^2)
\end{aligned}
$$

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&= \frac{1}{2} w_t^2 \cdot (1 - 2\tau + 2\tau^2 - \tau^2) \\
&= \frac{1}{2} w_t^2 \cdot (1 - 2\tau(1-\tau) - \tau^2) \\
&= \frac{1}{2} w_t^2 - \underbrace{\tau(1-\tau) w_t^2}_{=G_t(\tau)} - \underbrace{\frac{1}{2}\tau^2 w_t^2}_{:=DWL_t(\tau)}
\end{aligned}
$$

where $DWL_t(\tau)$ is the deadweight loss associated with the tax τ.

Deadweight Loss

- Note: The true "cost" of taxation is its distortionary effect on labor supply, leading to deadweight loss. While U_t^* falls by both $G_t(\tau)$ and $DWL_t(\tau)$, agents get the former back through government spending! The same is not true for deadweight loss.

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Deadweight Loss

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- Part f: When do increases in taxes generate more revenues than (deadweight) losses?
- Solution: We compute the partial derivatives

$$\begin{aligned}\frac{\partial}{\partial \tau} G_t(\tau) &= \frac{\partial}{\partial \tau}\left[w_t^2(\tau - \tau^2)\right] = w_t^2(1 - 2\tau) \\ \frac{\partial}{\partial \tau} DWL_t(\tau) &= \frac{\partial}{\partial \tau}\left[\frac{1}{2} w_t^2 \tau^2\right] = w_t^2 \tau\end{aligned}$$

and compare $w_t^2(1-2\tau) \stackrel{!}{=} w_t^2\tau \Leftrightarrow \tau_{\text{same}} = \frac{1}{3}$, so that increases in taxes when $\tau < \frac{1}{3}$ generates more revenues than losses!

Quiz 1

1. Imagine that in a given economy the level of capital stock per person is equal to 100 at time t. Furthermore, assume that the rate of depreciation of physical capital is equal to 7%, the rate of population growth is equal to 3%. How much should be invested at time t in this economy to keep the level of capital per person unchanged?
   - A 3
   - B 7
   - C 10 [x]
   - D 100

Quiz 1

2. Which of the following adjustments in a given economy should lead to a higher value of the steady state capital stock per person?
   A An improvement in technology
   B An increase in the saving rate
   C A reduction in the rate of depreciation
   D All of the above [x]

Quiz 1

3. Imagine that in a given Solow type economy the golden rule rate of saving is equal to 30%. Furthermore, imagine that currently the saving rate is equal to 20% and the economy happens to be in the steady state. We can be sure that an increase in the saving rate all the way to 30% would lead to
   - A an increase in welfare
   - B an increase in the rate of growth in the long run
   - C an increase in the level of output in the long run [x]
   - D a reduction in consumption in the long run

Quiz 1

4. Imagine that policy makers in a Solow type economy attempt to sustain growth in the long run by increasing the saving rate whenever the economy approaches the steady state. We can be sure that
   A can be successful as proven by the Soviet Union experience
   B must ultimately lead to a fall in the level of consumption [x]
   C can have a positive impact on the level of consumption in the long run, but not on the rate of growth of consumption
   D should lead to a reduction in the level of output in the long run as it will be too expensive to maintain a huge capital stock

5. Consider two economies. In the first output per person is given with $y = 10\sqrt{k}$ and in the second with $y = 20\sqrt{k}$. Furthermore, assume that the rate of growth of population and the depreciation of physical capital are the same in both economies. In addition, assume that the saving rate in the first economy is equal to 20%. If we want to ensure that the two economies enjoy the same rate of growth of output in the long then the saving rate in the second economy
   - A must be equal to 10%
   - B must be equal to 20%
   - C could assume any value [x]
   - D must be equal to 5%

Quiz 1

6. Imagine that in a given economy the saving rate is increased, but it remains below the golden rule of saving. We can be sure that
   A has a positive impact on the level output in the long run [x]
   B has a positive impact on the rate of growth of output in the long run
   C has a positive impact on the rate of growth of consumption in long run
   D has a positive impact on the level of consumption, but only in short run

Quiz 1

9. China in order to ensure that it will maintain a high rate of growth of output in the long-long run should
   - A reintroduce one-child policy
   - B abandon patent protection
   - C isolate itself from the rest of the world
   - D strengthen the enforcement of intellectual property rights [x]

Quiz 1

10. An increase in the saving rate
    A has a positive impact on output rate of growth in the long run in the Solow model
    B has a positive impact on output rate of growth in AK model [x]
    C does not affect the long run level of consumption in the Solow model
    D has only a transitory effect on output rate of growth in AK model

Quiz 1

11. Suppose that the level of output in a given economy can be represented with $Y_t = A_t K_t L_t^{\theta}$. Furthermore, assume that the rate of growth of variable X_t is denoted with g_X. What is the rate of growth of output in this economy?

    A $g_A + g_K$
    B $g_A + g_K + \theta g_L$ [x]
    C $g_A + g_K + g_L$
    D $g_A + g_K + (\theta - 1)g_L$

Quiz 1

12. Suppose that the level of output in a given economy can be represented with $Y_t = A_t K_t L_t^{\theta}$. The marginal product of capital is
    - A increasing
    - B constant [x]
    - C decreasing
    - D depends only on labor

Quiz 1

13. If the US wishes to overcome diminishing marginal product of capital then it should
    - A engage in more deficit spending
    - B increase human capital formation [x]
    - C increase its saving rate
    - D restrict capital outflows from the US

Quiz 1

14. Imagine that the production function in a Solow type economy is given with $Y_t = A_t\sqrt{K_t L_t}$. Assume that we know that TFP increases at 1% and that both capital and labor increase at 1% as well. What is the rate of increase of output per person in this economy?

    A 1% [x]
    B 2%
    C 3%
    D 0%

Quiz 1

15. Observed lack of convergence in the level of GDP per capita across the countries of the world can be attributed to
    - A the differences in the initial level of capital stock
    - B insufficient time as the convergence process is very slow
    - C differences in institutions across countries [x]
    - D the differences in the initial level of output per person