ECON 402 Discussion: Week 4 (problems)

Elird Haxhiu

University of Michigan

haxhiu@umich.edu

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Announcements

- Quiz 2 available today @ 4pm
- Topics: consumption, debt, and deficits

- Topics today
  1. Consumption problem
  2. Debt/deficit problem

Consumption

Example: an individual lives for $T = 3$ periods and derives utility from consumption $\{c_t\}_{t=1}^3$ according to the lifetime utility function

$$U = \sum_{t=1}^{3} \beta^{t-1} \frac{c_t^{1-\eta} - 1}{1 - \eta}$$

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where

- $\eta > 0$ is the coefficient of relative risk aversion (a higher value of η means the agent is more risk averse) $\Leftarrow$ see week2LEC.pdf for details!
- $\beta \in (0, 1)$ is the discount factor
- $r \geq 0$ is the real interest rate

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The agent starts life with $A_1 > 0$ assets, and receives income $\{y_t\}_{t=1}^3$

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The period budget constraints are

$$\begin{aligned} t = 1 &: \quad c_1 + s_1 = A_1 + y_1 \\ t = 2 &: \quad c_2 + s_2 = (1 + r)s_1 + y_2 \\ t = 3 &: \quad c_3 = (1 + r)s_2 + y_3 \end{aligned}$$

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and solving for s_1 and s_2 in the first two constrains and substituting into the third given the lifetime budget constraint

$$c_1 + \frac{1}{1+r}c_2 + \left(\frac{1}{1+r}\right)^2 c_3 = A_1 + y_1 + \frac{1}{1+r}y_2 + \left(\frac{1}{1+r}\right)^2 y_3$$

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\frac{\frac{1}{1-\eta}(1-\eta)c_1^{-\eta}}{\beta\frac{1}{1-\eta}(1-\eta)c_2^{-\eta}} &= \frac{\frac{1}{1+r}}{\left(\frac{1}{1+r}\right)^2}
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\Rightarrow \frac{c_2}{c_1} &= [\beta(1+r)]^{\frac{1}{\eta}} \\
\Rightarrow \frac{c_3}{c_2} &= [\beta(1+r)]^{\frac{1}{\eta}}
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c) Assume that $\beta = 1$ and $r = 0$. Use the two Euler equations and the budget constraint to solve for optimal consumption $\{c_t^*\}_{t=1}^3$.

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Under these assumptions, the three equations are

$$\begin{aligned} c_1 &= c_2 \\ c_2 &= c_3 \\ c_1 + c_2 + c_3 &= A_1 + y_1 + y_2 + y_3 \end{aligned}$$

which implies that $c_1^* = c_2^* = c_3^* = \frac{1}{3}(A_1 + y_1 + y_2 + y_3)$.

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d) Find optimal solutions in part c if income is constant: $y_t = \overline{y}$ for all t.

$$c_1^{**} = c_2^{**} = c_3^{**} = \frac{1}{3}(A_1 + 3\overline{y})$$

Consumption

e) Compute the partial derivative of your solutions in part d with respect to $\overline{y}$. Explain what they mean and compare their relative magnitudes.

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We have $\frac{\partial}{\partial \overline{y}} c_t^{**} = 1$ for all three time periods.

This makes sense because we're increasing income in every period, so the agent consumes one-for-one! Can you think about how (and more importantly, why) this answer would be different if $y_t \neq \overline{y}$ for all t and income y_t increases in only one time period?

Debts and Deficits

Example: suppose the economy lasts for $T = 3$ periods and there is a representative consumer whose lifetime utility function is given by

$$U = \sum_{t=1}^{3} \beta^{t-1} \left[\frac{C_t^{1-\eta} - 1}{1-\eta} - \frac{1}{2} L_t^2 \right]$$

where

- C_t is consumption
- $H := L_t + \ell_t$ are total hours in each period
- L_t is hours worked and ℓ_t is leisure time

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$\beta = 1$ so no discounting, wages $w_t = \overline{w}\ \forall t$, the interest rate $r_t = 0\ \forall t$, consumption price normalized to 1, and government spending is

$$G_t = t \cdot \overline{G}$$

in period t, where $\overline{G} > 0$.

Debts and Deficits

a) How much tax revenue should the government collect in each period?

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Since $r = 0$ and $\beta = 1$, we have $\beta(1 + r) = 1$ so we are in a steady-state and consumers will perfectly smooth their consumption over time: $C_1 = C_2 = C_3 = C_*$. This means that the government will also smooth the tax revenue it collects in each period, so that

$$T_* = T_1 = T_2 = T_3 = \frac{1}{3}(G_1 + G_2 + G_3) = \frac{\overline{G} + 2\overline{G} + 3\overline{G}}{3} = 2\overline{G}$$

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b) What is the optimal *deficit level* in period $t = 1$?

The deficit in the first period is simply the difference between government expenditures and tax revenues. Thus $D_1 = G_1 - T_1 = \overline{G} - 2\overline{G} = -\overline{G}$.

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The following table summarizes the deficit and debt in each period, and shows that the optimal debt level in period $t = 3$ is actually a saving of $\overline{G}$.

Period: t	1	2	3
Spending: G_t	$\overline{G}$	$2\overline{G}$	$3\overline{G}$
Tax Revenue: T_t	$2\overline{G}$	$2\overline{G}$	$2\overline{G}$
Deficit: $G_t - T_t$	$-\overline{G}$	0	$\overline{G}$
Debt: D_t	0	$-\overline{G}$	$-\overline{G}$

Debts and Deficits

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The consumer's utility function depends on both consumption and labor hours worked, where consumption is financed by working.

The period budget constraint is

$$C_t = \overline{w}L_t - \tau \cdot \overline{w}L_t = (1 - \tau)\overline{w}L_t$$

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The period budget constraint is

$$C_t = \overline{w}L_t - \tau \cdot \overline{w}L_t = (1 - \tau)\overline{w}L_t$$

which we can substitute into the lifetime utility function. This gives

$$U(L_t) = \frac{[(1 - \tau)\overline{w}L_t]^{1-\eta} - 1}{1 - \eta} - \frac{1}{2}L_t^2$$

The first-order condition for labor supply in period t is

$$\frac{\partial}{\partial L_t} U(L_t) = 0$$

Debts and Deficits

The first-order condition for labor supply in period t is

$$\begin{aligned}\frac{\partial}{\partial L_t} U(L_t) &= 0 \\ \left[(1-\tau)\overline{w}\right]^{1-\eta} L_t^{-\eta} - L_t &= 0 \\ L_t^* &= \left[(1-\tau)\overline{w}\right]^{\frac{1-\eta}{1+\eta}}\end{aligned}$$

which gives optimal labor supply.