ECON 402 Discussion: Week 4

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Announcements

- Homework 1 solutions posted
- Homework 2 due Friday February 3rd @ 6pm

- Exam 1 on Wednesday February 8th @ 6pm (2 locations, check Canvas)
- Topics = macro data, general equilibrium (GE) model, and Solow model

- Topics today
  1. Recap basic GE model
  2. Golden rule saving rate
  3. Solow model with technology + population growth
  4. Accounting for growth with Cobb-Douglas

Neoclassical Production Functions $Y = F(K, L)$

1. [Continuity] — F is continuous and twice differentiable
2. [Marginal Products > 0] — $F_K := \frac{\partial}{\partial K} F > 0$ and $F_L := \frac{\partial}{\partial L} F > 0$
3. [MPs diminishing] — $F_{KK} := \frac{\partial^2}{\partial K^2} F < 0$ and $F_{LL} := \frac{\partial^2}{\partial L^2} F < 0$
4. [Constant Returns to Scale] — For all $\lambda > 0$, we have $F(\lambda \cdot K, \lambda \cdot L) = \lambda \cdot F(K, L)$
5. [Factor Complementarity] — $\frac{\partial}{\partial K} MPL > 0$ and $\frac{\partial}{\partial L} MPK > 0$

“Short-run” General Equilibrium (GE) model

4 equations for 4 endogenous variables: Y, C, I, r

$$\begin{aligned} Y &= F(K, L) \\ Y &= C + I + G \\ C &= C(Y - T, r) \\ I &= I(r) \end{aligned}$$

Market for loan-able funds = another way of expressing the goods market clearing condition!

$$\begin{aligned} S(r) &= Y - C - G \\ &= Y - C(Y - T, r) - G \\ &= F(K, L) - C(F(K, L) - T, r) - G \end{aligned}$$

Note that aggregate (desired) savings depend positively on the interest rate while aggregate (desired) investment $I(r)$ depends negatively on $r \Rightarrow$ unique solution r^*

Predictions from the GE model

Shifting curves and changing equilibrium given *exogenous* shocks to economy...

EX1: Contrast the effects of immigration shocks on labor vs capital markets.
EX2: What does technological innovation do to short-run interest rates?
EX3: How does government spending via borrowing affect availability of loan-able funds?

See homework 1 solutions for details!

Basic Solow model of economic growth

- Accounting: $Y_t = C_t + I_t$ with $G_t = NX_t = 0$
- Production: $Y_t = K_t^\alpha L_t^{1-\alpha}$ with $\alpha \in (0,1)$
- Input prices: $R_t = MPK_t$ and $w_t = MPL_t$

- Behavioral assumption about saving
  - $I_t = s \cdot Y_t$ where $s \in (0,1)$ is exogenous
  - $C_t = (1-s) \cdot Y_t$

- Law of motion for capital input
  - $\Delta K_t = I_t - \delta \cdot K_t$ where $\delta \in (0,1)$

- Per capita quantities: $k_t = \frac{K_t}{L_t}$, $y_t = \frac{Y_t}{L_t}$, and $c_t = \frac{C_t}{L_t}$

Example: The Golden Rule

What level of saving maximizes consumption per capita in steady state ($\Delta k_t = 0$)?

1. Find the law of motion for the capital-labor ratio k_t
2. Find the steady state capital-labor ratio k_* where $\Delta k_t = 0$
3. Find consumption per capita in steady state c_*

4. Prove that with Cobb-Douglas production, the optimal saving rate is equal to $\alpha \in (0,1)$ by solving the first-order condition (FOC) $\frac{\partial}{\partial s} c_*(s) = 0$ for optimal s

5. Show graphically that the optimal saving rate implies $MPK = \delta$. Hint: in steady-state we must have $sf(k_t) = \delta k_t$ so that $c_t := (1-s)y_t = f(k_t) - \delta k_t$.

Solow model with technology and population growth

- Accounting: $Y_t = C_t + I_t$ with $G_t = NX_t = 0$
- Production, with labor-augmenting technology: $Y_t = K_t^{\alpha}(E_t \cdot L_t)^{1-\alpha}$ with $\alpha \in (0,1)$
- Input prices: $R_t = MPK_t$ and $w_t = MPL_t$

- Behavioral assumption about saving
  - $I_t = s \cdot Y_t$ where $s \in (0,1)$ is exogenous
  - $C_t = (1-s) \cdot Y_t$

- Laws of motion for capital, labor, and technical progress
  - $\Delta K_t = I_t - \delta \cdot K_t$ where $\delta \in (0,1)$
  - $L_{t+1} = (1+n) \cdot L_t > 0$ for all t
  - $E_{t+1} = (1+g) \cdot E_t > 0$ for all t

- Per capita quantities, intensive form: $k_t = \frac{K_t}{E_t \cdot L_t}$, $y_t = \frac{Y_t}{E_t \cdot L_t}$, and $c_t = \frac{C_t}{E_t \cdot L_t}$

Golden rule with technology and population growth

Allowing for technical progress ($g > 0$) and positive population growth ($n > 0$), what level of saving maximizes consumption per capita in steady state ($\Delta k_t = 0$)?

1. Find the law of motion for the intensive form capital-labor ratio k_t
2. Find the steady state intensive form capital-labor ratio k_* where $\Delta k_t = 0$
3. Find golden rule saving rate implied by the $MPK = \delta$ optimality condition. How does this optimal saving rate compare to the case where $g = n = 0$?

Technical progress = growth not due to K or L

Solow model does not explain where technical progress comes from; only makes predictions for other macro variables based on the assumption it exists... ("black box" approach)

Under constant returns to scale and perfect competition, consider small changes Δ

$$\begin{aligned}
Y &= MPK \cdot K + MPL \cdot L \\
\frac{\Delta Y}{Y} &= MPK \cdot \frac{\Delta K}{Y} + MPL \cdot \frac{\Delta L}{Y} \\
\frac{\Delta Y}{Y} &= \frac{MPK \cdot K}{Y} \cdot \frac{\Delta K}{K} + \frac{MPL \cdot L}{Y} \cdot \frac{\Delta L}{L} \\
g_Y &= \frac{MPK \cdot K}{Y} \cdot g_K + \frac{MPL \cdot L}{Y} \cdot g_L \\
g_Y &= \alpha g_K + (1-\alpha) g_L
\end{aligned}$$

where $\alpha \in (0,1)$ denotes the capital share (same as Cobb-Douglas specification $Y = K^{\alpha} L^{1-\alpha}$)

Technical progress = growth not due to K or L

Define technical growth as excess growth in output not due to growth in machines or people

$$
\begin{aligned}
g_Y &= \alpha g_K + (1-\alpha) g_L \\
g_A &:= g_Y - \alpha g_K - (1-\alpha) g_L
\end{aligned}
$$

to the extent that true equality does not hold in real data

Some important things I want you to know

1. This definition hints at the weird Neoclassical idea that technology can also regress (α-weighted growth in K and L exceeds growth in Y) as in the RBC model... stay tuned!
2. This formulation assumes "factor neutral" technology in the production function such as $Y = A \cdot K^{\alpha} L^{1-\alpha}$ instead of "labor augmenting" like we specified in the Solow model $Y = K^{\alpha}(E \cdot L)^{1-\alpha}$, but this changes nothing