ECON 402 Discussion: Week 5 (problems)

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Announcements

- Topics today
  1. Introduction
  2. Open economy IS-LM
  3. Real business cycle (RBC) model

Introduction

- We studied short and long run equilibria in the IS-LM and AS-AD framework and the role for government policy in responding to fluctations (aka recessions).
- Today we extend this task in two important directions.

- First, we consider these "old school" macro models in the context of a small open economy (SOE), where the interest rate is fixed.

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- Today we extend this task in two important directions.

- First, we consider these “old school” macro models in the context of a small open economy (SOE), where the interest rate is fixed.
- Second, we return to our “grown up” macro models and consider implications of adding labor supply to the dynamic consumption decision. The result is the famous RBC model...

- Things will get weird because although RBC provides a clear explanation for recessions, it implies no role for government policy...

Open Economy

- Example: Solve the open economy model described by
  - Money demand: $\left(\frac{M}{P}\right)_D = Y_t - 1000r_t$.
  - Money supply: $\left(\frac{M}{P}\right)_S = 1000$.

  - Investment demand: $I(r_t) = 200 - 1000r_t$.
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  - Investment demand: $I(r_t) = 200 - 1000r_t$.
  - Consumption: $C_t = 0.6(Y_t - \overline{T})$.
  - Net export demand: $NX(e_t) = 100 + 100e_t$, where

$$e_t := \frac{\$_{US}}{\$_{FOR}}$$

    is the nominal exchange rate. Why is $NX'(e_t) > 0$?

  - Government: $\overline{G} = \overline{T} = 0$
  - World interest rate: $r^* = 0.05$

Open Economy

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or $e_t = \frac{1}{250}Y_t + \frac{625}{250}$ for graphing!

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- Part b: Derive the LM curve.
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$$\begin{aligned} \left(\frac{M}{P}\right)_D &\stackrel{!}{=} \left(\frac{M}{P}\right)_S \\ 1000 &= Y_t - 1000(0.05) \\ Y_t &= 1050 \end{aligned}$$

- Part c: Find the short run equilibrium.
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Open Economy

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which implies that

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- Note: This implies that the exchange rate depreciates in response to government spending, which reduces (net) exports.

RBC Model

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  1. Practical: stagflation in the 1970s ($Y_t \downarrow$ and $\pi_t \uparrow$).
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- The Real Business Cycle (RBC) model...
  - Corrects these problems (somewhat).
  - Explains economic fluctuations with total factor productivity (TFP) shocks (aka shocks to technology A_t in production function).
  - Assumes competitive markets $\Rightarrow$ first welfare theorem (FWT) holds $\Rightarrow$ inactive policy recommendations are baked in...
  - Note: "real" means that all variables are in consumption units, so there's no role for money or nominal variables!

RBC Model

Some important business cycle facts:

	US data		RBC model	
$\heartsuit$	st dev	corr($\heartsuit$, Y_t)	st dev	corr($\heartsuit$, Y_t)
Y_t	1.7	1		
C_t	0.8	0.7		
I_t	8.2	0.9		
L_t	1.6	0.8		
A_t	-	-		

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$$\begin{aligned} \max_{C_0, C_1, L_0, L_1, S_1} \quad & \sum_{t=1}^{2} \beta^{t-1} [\log C_t + \log(1 - L_t)] \\ \text{s.t.} \quad & C_0 + S_1 = w_0 L_0 \\ & C_1 = w_1 L_1 + (1 + r) S_1 \end{aligned}$$

RBC Model

- Solution: After deriving the lifetime budget constraint (you should be able to do this...), the Lagrangian is given by

$$\mathscr{L} = \sum_{t=1}^{2} \beta^{t-1}[\log C_t + \log(1 - L_t)] + \lambda \left[w_0 L_0 + \frac{1}{1+r} w_1 L_1 - C_0 - \frac{1}{1+r} C_1 \right]$$

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and the first order conditions (FOCs) are

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\mathscr{L}_{C_0} &: \quad \frac{1}{C_0} - \lambda = 0 \\
\mathscr{L}_{C_1} &: \quad \beta \frac{1}{C_1} + \lambda \left(-\frac{1}{1+r} \right) = 0 \\
\mathscr{L}_{L_0} &: \quad \frac{-1}{1 - L_0} + w_0 \lambda = 0 \\
\mathscr{L}_{L_1} &: \quad \beta \frac{-1}{1 - L_1} + \lambda w_1 \frac{1}{1+r} = 0
\end{aligned}$$

RBC Model

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Substituting into the last two FOCs implies that

$$\begin{aligned} L_0 &= 1 - \frac{C_0}{w_0} \\ L_1 &= 1 - \frac{\beta(1+r)C_0}{w_1} \end{aligned}$$

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Combining our FOCs in this way means we can write C_1, L_0, and L_1 in terms of C_0. Substituting all of these into the lifetime budget constraint gives us the solution for period 0 consumption

$$C_0^* = \frac{w_0 + \frac{1}{1+r} w_1}{2(1 + \beta)}$$

RBC Model

- Solution: Substituting this into our previous expressions gives

$$
\begin{aligned}
C_0^* &= \frac{w_0 + \frac{1}{1+r} w_1}{2(1+\beta)} \\
C_1^* &= \beta(1+r)\frac{w_0 + \frac{1}{1+r} w_1}{2(1+\beta)} \\
L_0^* &= 1 - \frac{1}{w_0}\frac{w_0 + \frac{1}{1+r} w_1}{2(1+\beta)} \\
L_1^* &= 1 - \frac{\beta(1+r)}{w_1}\frac{w_0 + \frac{1}{1+r} w_1}{2(1+\beta)}
\end{aligned}
$$

while saving (or borrowing) is given by

$$S_1^* = w_0 L_0^* - C_0^*$$

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\begin{aligned}
\pi(K_t, L_t) &= PF(K_t, L_t) - R_t K_t - w_t L_t \\
&= A_t K_t^{\alpha} L_t^{1-\alpha} - R_t K_t - w_t L_t
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and their FOCs for optimization imply that

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so RBC implies factor prices are "pro-cyclical." Given our solutions

$$\{C_0^*, C_1^*, L_0^*, L_1^*, S_1^*\}$$

in step 1, we see that consumption is also pro-cyclical since it depends positively on wages (you should verify this...).

The effect on hours is ambiguous due to income/substitution effects.

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L_t	1.6	0.8	0.7	0.98
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