ECON 402 Exam 1 Review

Elird Haxhiu

University of Michigan

haxhiu@umich.edu

February 7, 2023

Neoclassical Production Functions $Y = F(K, L; A)$

1. [Continuity] F is continuous and twice differentiable ✓

2. [Marginal Products > 0] $MPK := \frac{\partial}{\partial K} F > 0$ and $MPL := \frac{\partial}{\partial L} F > 0$

Neoclassical Production Functions $Y = F(K, L; A)$

1. [Continuity] F is continuous and twice differentiable
2. [Marginal Products > 0] $MPK := \frac{\partial}{\partial K} F > 0$ and $MPL := \frac{\partial}{\partial L} F > 0$
3. [MPs diminishing] $\frac{\partial}{\partial K} MPK := \frac{\partial^2}{\partial K^2} F < 0$ and $\frac{\partial}{\partial L} MPL := \frac{\partial^2}{\partial L^2} F < 0$

Neoclassical Production Functions $Y = F(K, L; A)$

1. [Continuity] F is continuous and twice differentiable
2. [Marginal Products > 0] $MPK := \frac{\partial}{\partial K} F > 0$ and $MPL := \frac{\partial}{\partial L} F > 0$
3. [MPs diminishing] $\frac{\partial}{\partial K} MPK := \frac{\partial^2}{\partial K^2} F < 0$ and $\frac{\partial}{\partial L} MPL := \frac{\partial^2}{\partial L^2} F < 0$
4. [Factor Complementarity] $\frac{\partial}{\partial K} MPL > 0$ and $\frac{\partial}{\partial L} MPK > 0$

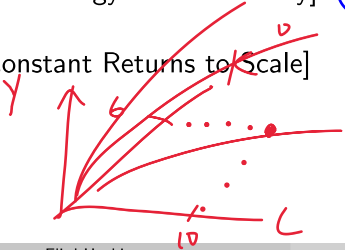
Neoclassical Production Functions $Y = F(K, L, A)$ $= A \cdot K^{\alpha} L^{1-\alpha}$ $\alpha \in (0,1)$

1. [Continuity] — F is continuous and twice differentiable $= K^{\alpha}(A \cdot L)^{1-\alpha}$
2. [Marginal Products > 0] — $MPK := \frac{\partial}{\partial K} F > 0$ and $MPL := \frac{\partial}{\partial L} F > 0$ $= A^{1-\alpha} \cdot K^{\alpha} L^{-\alpha}$
3. [MPs diminishing] — $\frac{\partial}{\partial K} MPK := \frac{\partial^2}{\partial K^2} F < 0$ and $\frac{\partial}{\partial L} MPL := \frac{\partial^2}{\partial L^2} F < 0$
4. [Factor Complementarity] — $\frac{\partial}{\partial K} MPL > 0$ and $\frac{\partial}{\partial L} MPK > 0$
5. [Technology & Productivity] — $\frac{\partial}{\partial A} MPL > 0$ and $\frac{\partial}{\partial A} MPK > 0$

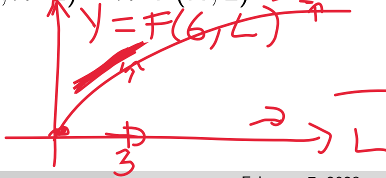
Neoclassical Production Functions $Y = F(K, L; A)$

1. [Continuity] — F is continuous and twice differentiable
2. [Marginal Products > 0] — $MPK := \frac{\partial}{\partial K} F > 0$ and $MPL := \frac{\partial}{\partial L} F > 0$
3. [MPs diminishing] — $\frac{\partial}{\partial K} MPK := \frac{\partial^2}{\partial K^2} F < 0$ and $\frac{\partial}{\partial L} MPL := \frac{\partial^2}{\partial L^2} F < 0$

4. [Factor Complementarity] — $\frac{\partial}{\partial K} MPL > 0$ and $\frac{\partial}{\partial L} MPK > 0$
5. [Technology & Productivity] — $\frac{\partial}{\partial A} MPL > 0$ and $\frac{\partial}{\partial A} MPK > 0$
6. [Constant Returns to Scale] — $\forall \lambda > 0 \Rightarrow F(\lambda \cdot K, \lambda \cdot L) = \lambda \cdot F(K, L)$



$\bar{K} = 6$



Static, Short-run General Equilibrium (GE) model

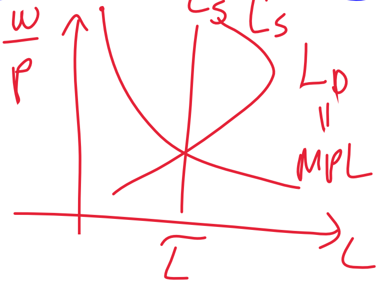
SE(0,1)

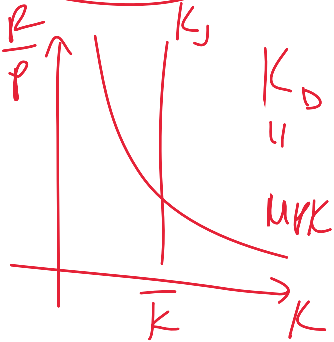
4 equations, 4 endogenous (Y, C, I, r) & 5 exogenous ($A, \overline{L}, \overline{K}, G, T$) vars, 2 parameters ($\delta, \theta$)

4 equations, 4 endogenous (Y, C, I, r) & 5 exogenous $(A, \overline{L}, \overline{K}, G, T)$ vars, 2 parameters (δ, θ)

$$Y = A \cdot F(K, L) \qquad \frac{w}{P} = MPL(A, K) \qquad \frac{R}{P} = MPK(A, L)$$

$$\max_{K,L} P \cdot A F(K, L) - R \cdot K - w \cdot L$$





Static, Short-run General Equilibrium (GE) model

4 equations, 4 endogenous (Y, C, I, r) & 5 exogenous $(A, \overline{L}, \overline{K}, G, T)$ vars, 2 parameters (δ, θ)

$$\begin{aligned} Y &= A \cdot F(K, L) \\ C &= C(\underset{+}{Y - T}, \underset{-}{r}) \end{aligned}$$

$$\frac{w}{P} = MPL(A, K) \qquad \frac{R}{P} = MPK(A, L)$$

example: $C = \theta(Y - T) - r$

$$\frac{\partial}{\partial [Y-T]} C = \theta$$

$$< 1$$

Static, Short-run General Equilibrium (GE) model

4 equations, 4 endogenous (Y, C, I, r) & 5 exogenous $(A, \overline{L}, \overline{K}, G, T)$ vars, 2 parameters (δ, θ)

$K = X \cdot P_K$

$$\frac{w}{P} = MPL(A, K) \qquad \frac{R}{P} = MPK(A, L)$$

$$\begin{aligned} Y &= A \cdot F(K, L) \\ C &= C(Y - T, r) \\ I &= I(r, MPK, \delta) \end{aligned}$$

example: $C = \theta(Y - T) - r$

from $\frac{P \cdot MPK}{P} - \delta \cdot \frac{P_K}{P} = \frac{r \cdot P_K}{P}$ $\leftarrow 1?$

$- \quad + \quad -$

$\max_I \; P_K \cdot x(I) - I$

$\Rightarrow I^* = f(\underset{+}{P_K})$

$I = f\left(\underset{+}{\frac{P_K}{P}}\right)$

$MPK - \delta \cdot \frac{P_K}{P} = r \cdot \frac{P_K}{P}$

$$\boxed{\frac{MPK}{r+\delta} = \frac{P_K}{P}}$$

$\cancel{[r+\delta]}$

Static, Short-run General Equilibrium (GE) model

4 equations, 4 endogenous (Y, C, I, r) & 5 exogenous $(A, \overline{L}, \overline{K}, G, T)$ vars, 2 parameters (δ, θ)

$$
\begin{aligned}
Y &= A \cdot F(K, L) \\
C &= C(Y - T, r) \\
I &= I(r, MPK, \delta) \\
Y &\overset{!}{=} C + I + G
\end{aligned}
$$

$$\frac{w}{P} = MPL(A, K) \qquad \frac{R}{P} = MPK(A, L)$$

example: $C = \theta(Y - T) - r$

from $P \cdot MPK - \delta \cdot P_K = r \cdot P_K$

$$\Rightarrow \underbrace{Y - C - G}_{S(r)} = I$$

$$S(r) \overset{!}{=} I(r)$$

Static, Short-run General Equilibrium (GE) model

4 equations, 4 endogenous (Y, C, I, r) & 5 exogenous $(A, \overline{L}, \overline{K}, G, T)$ vars, 2 parameters (δ, θ)

$$\begin{aligned} Y &= A \cdot F(K, L) \\ C &= C(Y - T, r) \\ I &= I(r, MPK, \delta) \\ Y &= C + I + G \end{aligned}$$

$$\frac{w}{P} = MPL(A, K) \qquad \frac{R}{P} = MPK(A, L)$$

example: $C = \theta(Y - T) - r$

from $P \cdot MPK - \delta \cdot P_K = r \cdot P_K$

Aggregate (desired) investment $I(r)$ depends negatively on r while aggregate (desired) savings

$$\begin{aligned} S(r) &= Y - C - G \\ &= F(K, L) - C(F(K, L) - T, r) - G \end{aligned}$$

4 equations, 4 endogenous (Y, C, I, r) & 5 exogenous $(A, \bar{L}, \bar{K}, G, T)$ vars, 2 parameters (δ, θ)

Ls Ks

$MPL = Lp$ MPK

$$
\begin{aligned}
Y &= A \cdot F(K, L) & \frac{w}{P} &= MPL(A, K) & \frac{R}{P} &= MPK(A, L) \\
C &= C(Y - T, r) & \text{example:} \quad & C = \theta(Y - T) - r \\
I &= I(r, MPK, \delta) & \text{from} \quad & P \cdot MPK - \delta \cdot P_K = r \cdot P_K \\
Y &= C + I + G
\end{aligned}
$$

$S(r) = I(r)$

$S(r)$ $I(r)$ r^*

Aggregate (desired) investment $I(r)$ depends negatively on r while aggregate (desired) savings

$$S(r) = Y - C - G$$

$$\boxed{S(r) = F(K, L) - C(F(K, L) - T, r) - G}$$

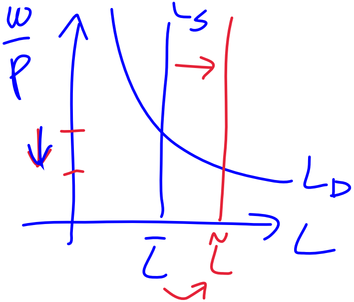
depends positively on the interest rate $\Rightarrow$ unique solution r^* in market for loan-able funds!

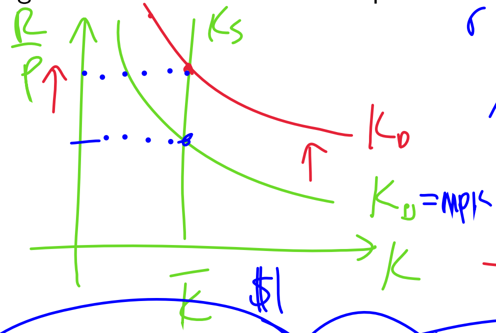
Predictions from the GE model

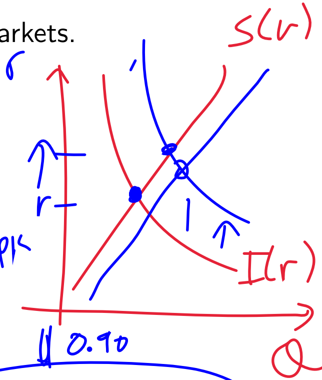
$\theta < 1$

Shifting curves and changing equilibrium given *exogenous* shocks to economy...

EX1: Contrast the effects of immigration shocks on labor vs capital markets.







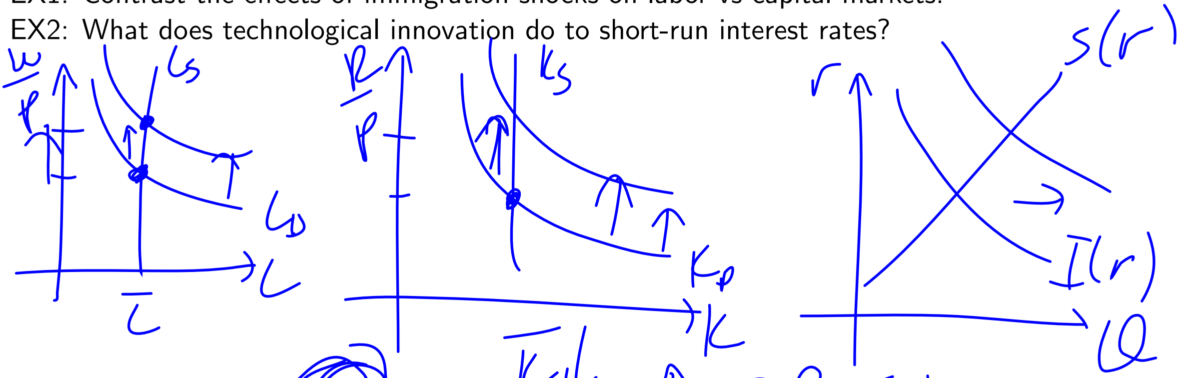
$$L \uparrow \Rightarrow Y = F(K, L) \uparrow \rightarrow C = C(Y - T, r) \uparrow$$

Predictions from the GE model

$Y = R \cdot K + w \cdot L$

Shifting curves and changing equilibrium given *exogenous* shocks to economy...

EX1: Contrast the effects of immigration shocks on labor vs capital markets.
EX2: What does technological innovation do to short-run interest rates?



$A\uparrow \Rightarrow Y\uparrow$ $\quad C = C(Y - T, r)\uparrow$ $\quad \bar{K}\downarrow$ $\quad Q?$ $\quad C\uparrow$

Predictions from the GE model

Shifting curves and changing equilibrium given *exogenous* shocks to economy...

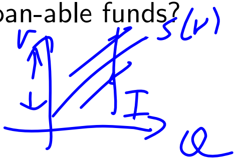
EX1: Contrast the effects of immigration shocks on labor vs capital markets.
EX2: What does technological innovation do to short-run interest rates?
EX3: How does government spending via borrowing affect availability of loan-able funds?

Predictions from the GE model

Shifting curves and changing equilibrium given *exogenous* shocks to economy...

EX1: Contrast the effects of immigration shocks on labor vs capital markets.
EX2: What does technological innovation do to short-run interest rates?
EX3: How does government spending via borrowing affect availability of loan-able funds?
EX4: REVIEW QUESTION GE - long answer

$I = I(MPK)$

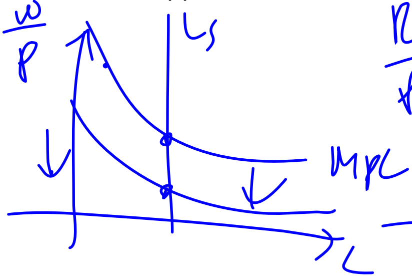


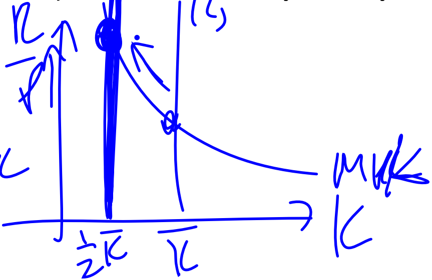
Predictions from the GE model

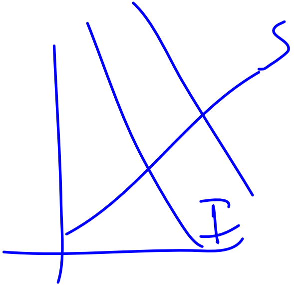
Shifting curves and changing equilibrium given *exogenous* shocks to economy...

EX1: Contrast the effects of immigration shocks on labor vs capital markets.
EX2: What does technological innovation do to short-run interest rates?
EX3: How does government spending via borrowing affect availability of loan-able funds?
EX4: REVIEW QUESTION GE - long answer
EX5: What happens if half of the capital stock suddenly destroyed?

G↑







Solow model with technology and population growth (dynamic!)

- Accounting: $Y_t = C_t + I_t$ with $G_t = NX_t = 0$

$S = Y - C$

Solow model with technology and population growth

- Accounting: $Y_t = C_t + I_t$ with $G_t = NX_t = 0$
- Production, with labor-augmenting technology: $Y_t = K_t^{\alpha}(E_t \cdot L_t)^{1-\alpha}$ with $\alpha \in (0,1)$

Solow model with technology and population growth

- Accounting: $Y_t = C_t + I_t$ with $G_t = NX_t = 0$
- Production, with labor-augmenting technology: $Y_t = K_t^{\alpha}(E_t \cdot L_t)^{1-\alpha}$ with $\alpha \in (0,1)$
- Input prices: $R_t = MPK_t$ and $w_t = MPL_t$

Solow model with technology and population growth

- Accounting: $Y_t = C_t + I_t$ with $G_t = NX_t = 0$
- Production, with labor-augmenting technology: $Y_t = K_t^{\alpha}(E_t \cdot L_t)^{1-\alpha}$ with $\alpha \in (0, 1)$
- Input prices: $R_t = MPK_t$ and $w_t = MPL_t$

- Behavioral assumption about saving
  - $I_t = s \cdot Y_t$ where $s \in (0, 1)$ is exogenous
  - $C_t = (1 - s) \cdot Y_t$

GE: $I_t = I(r_t, MPK_t, \delta)$

$C_t = c(Y_t - T_t, r_t)$

Solow model with technology and population growth

$x(I_t)$? $< K$

- Accounting: $Y_t = C_t + I_t$ with $G_t = NX_t = 0$
- Production, with labor-augmenting technology: $Y_t = K_t^{\alpha}(E_t \cdot L_t)^{1-\alpha}$ with $\alpha \in (0,1)$
- Input prices: $R_t = MPK_t$ and $w_t = MPL_t$

$$\boxed{K_{t+1} = K_t(1-\delta) + I_t}$$

1-for-1

- Behavioral assumption about saving

(1)
  - $I_t = s \cdot Y_t$ where $s \in (0,1)$ is exogenous
  - $C_t = (1-s) \cdot Y_t$

$$\underbrace{K_{t+1} - K_t}_{} = I_t - \delta K_t$$

- Laws of motion for capital, labor, and technical progress

(2)
  - $\Delta K_t = I_t - \delta \cdot K_t$ where $\delta \in (0,1)$

$$\boxed{\Delta K = I - \delta K}$$

Solow model with technology and population growth

- Accounting: $Y_t = C_t + I_t$ with $G_t = NX_t = 0$
- Production, with labor-augmenting technology: $Y_t = K_t^{\alpha}(E_t \cdot L_t)^{1-\alpha}$ with $\alpha \in (0,1)$
- Input prices: $R_t = MPK_t$ and $w_t = MPL_t$

- Behavioral assumption about saving
  - $I_t = s \cdot Y_t$ where $s \in (0,1)$ is exogenous
  - $C_t = (1-s) \cdot Y_t$

- Laws of motion for capital, labor, and technical progress
  - $\Delta K_t = I_t - \delta \cdot K_t$ where $\delta \in (0,1)$
  - $L_{t+1} = (1+n) \cdot L_t > 0$ for all t
  - $E_{t+1} = (1+g) \cdot E_t > 0$ for all t

$L_{t+1} = L_t + nL_t$

$\frac{\Delta L}{L} = n$

$\frac{\Delta L}{L} = n$

$\frac{\Delta E}{E} = g$

Solow model with technology and population growth

- Accounting: $Y_t = C_t + I_t$ with $G_t = NX_t = 0$
- Production, with labor-augmenting technology: $Y_t = K_t^{\alpha}(E_t \cdot L_t)^{1-\alpha}$ with $\alpha \in (0,1)$
- Input prices: $R_t = MPK_t$ and $w_t = MPL_t$

- Behavioral assumption about saving
  - $I_t = s \cdot Y_t$ where $s \in (0,1)$ is exogenous
  - $C_t = (1-s) \cdot Y_t$

- Laws of motion for capital, labor, and technical progress
  - $\Delta K_t = I_t - \delta \cdot K_t$ where $\delta \in (0,1)$
  - $L_{t+1} = (1+n) \cdot L_t > 0$ for all t
  - $E_{t+1} = (1+g) \cdot E_t > 0$ for all t

- Per capita quantities, intensive form: $k_t = \frac{K_t}{E_t \cdot L_t}$, $y_t = \frac{Y_t}{E_t \cdot L_t}$, and $c_t = \frac{C_t}{E_t \cdot L_t}$

Cobb-Douglas k_t^{α}

CES $= f(k_t) =$

$$\Delta K = I - \delta \cdot K$$

Allowing for technical progress ($g > 0$) and population growth ($n > 0$), what level of saving maximizes consumption per capita in steady state ($\Delta k_t = 0$)?

$$\Delta k = s f(k) - (\delta + g + n) k$$

1. Find the law of motion for the intensive form capital-labor ratio k_t

$$k := \frac{K}{E \cdot L} \Rightarrow \frac{\Delta k}{k} = \frac{\Delta K}{K} - \frac{\Delta(E \cdot L)}{E \cdot L}$$

$$= \frac{\Delta K}{K} - \left(\frac{\Delta E}{E} + \frac{\Delta L}{L} \right)$$

$$= \frac{I - \delta K}{K} - (g + n)$$

$$\frac{\Delta k}{k} = \frac{s \cdot Y}{k} k - (\delta + g + n) k$$

$$\Delta k = \frac{s \cdot Y}{\cancel{k}} \frac{\cancel{K}}{E \cdot L} - (\delta + g + n) k$$

$$\Delta k = s \cdot f(k_t) - (\delta + g + n) k$$

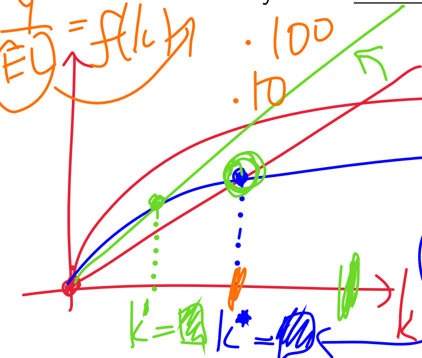
Golden rule with technology and population growth

$$\Delta k = s \cdot k^{\alpha} - (\delta + g + n)k \overset{!}{=} 0$$

Allowing for technical progress ($g > 0$) and population growth ($n > 0$), what level of saving maximizes consumption per capita in steady state ($\Delta k_t = 0$)?

$$Y = C + I + G \qquad A \cdot \qquad Y = y \cdot L$$

1. Find the law of motion for the intensive form capital-labor ratio k_t
2. Find the steady state intensive form capital-labor ratio k_* where $\Delta k_t = 0$



$$0 \overset{!}{=} s k^{\alpha} - (\delta + g + n)k$$

$$\frac{(\delta+g+n)k}{k} = \frac{s k^{\alpha}}{k}$$

$$\left(\frac{\delta+g+n}{s}\right)^{\frac{1}{\alpha-1}} = k$$

$$k^* = \left(\frac{s}{\delta+g+n}\right)^{\frac{1}{1-\alpha}}$$

0.9

$$C = \theta(Y - T)$$

Golden rule with technology and population growth

$f(k) = k^\alpha$

$MPK = f'(k)$

$MPK = \frac{\partial F}{\partial K}$

Allowing for technical progress ($g > 0$) and population growth ($n > 0$), what level of saving maximizes consumption per capita in steady state ($\Delta k_t = 0$)?

1. Find the law of motion for the intensive form capital-labor ratio k_t
2. Find the steady state intensive form capital-labor ratio k_* where $\Delta k_t = 0$
3. Find golden rule saving rate implied by the $MPK = \delta$ optimality condition. How does this optimal saving rate compare to the case where $g = n = 0$?

$\boxed{\max_s (1-s) \cdot f(k)} \iff \max_s f(k) - sf(k) \iff \max_s f(k) - (\delta+g+n)k$

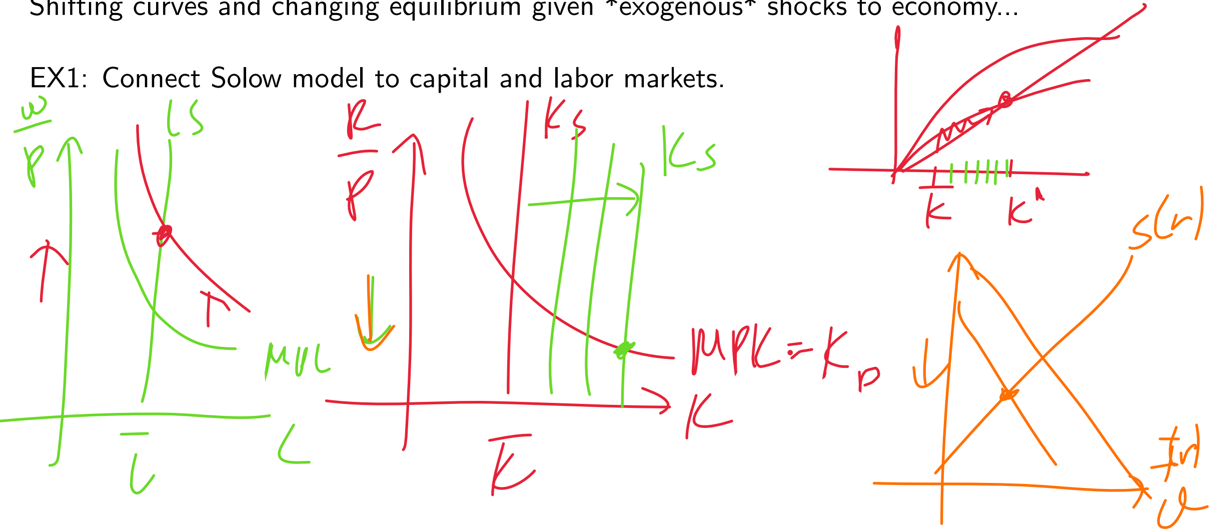
$f'(k) = \delta + g + n$

$\alpha k^{\alpha-1} = \delta + g + n$

$\boxed{s^G = \alpha}$

Shifting curves and changing equilibrium given *exogenous* shocks to economy...

EX1: Connect Solow model to capital and labor markets.



Predictions from the Solow model

Shifting curves and changing equilibrium given *exogenous* shocks to economy...

EX1: Connect Solow model to capital and labor markets.
EX2: What happens during capital accumulation, or transition to steady state?

Predictions from the Solow model

Shifting curves and changing equilibrium given *exogenous* shocks to economy...

EX1: Connect Solow model to capital and labor markets.
EX2: What happens during capital accumulation, or transition to steady state?
EX3: What happens in a steady state with population growth?

$K\uparrow \ldots$

Predictions from the Solow model

Shifting curves and changing equilibrium given *exogenous* shocks to economy...

EX1: Connect Solow model to capital and labor markets.
EX2: What happens during capital accumulation, or transition to steady state?
EX3: What happens in a steady state with population growth?
EX4: REVIEW QUESTIONS SOLOW - long answer

Thanks for your attention!

And good luck tomorrow!