ECON 402 Discussion: Week 6 (PROB)

Elird Haxhiu

University of Michigan

haxhiu@umich.edu

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Announcements

- Quiz 3 available today starting at 4pm.
- Optional/extra credit HW released today, due next Friday!
- Submission: written/typed answers uploaded to Canvas.

- Topics today
    1. DSGE question
    2. Open Economy question

DSGE question

Suppose output, the price level, and money in an economy are given by

$$
\begin{aligned}
y_t &= \overline{y} + b(m_t - E_{t-1}(m_t)) \\
p_t &= E_{t-1}(m_t) + (1-b)(m_t - E_{t-1}(m_t)) \\
m_t &= \overline{\lambda} + \rho_0 \varepsilon_t + \rho_1 \varepsilon_{t-1}
\end{aligned}
$$

where y_t is output, p_t is the price level, m_t is money, and $\overline{y}$, $\overline{\lambda}$, ρ_0, ρ_1, and b are constants. Lower case variables are in logs, and the shocks ε_t are mean zero and independent over time.

DSGE question

a) Find the level of output in this economy.

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We first find the expected value of money in this economy

$$\begin{aligned} E_{t-1}(m_t) &= E_{t-1}(\overline{\lambda} + \rho_0 \varepsilon_t + \rho_1 \varepsilon_{t-1}) \\ &= E_{t-1}(\overline{\lambda}) + E_{t-1}(\rho_0 \varepsilon_t) + E_{t-1}(\rho_1 \varepsilon_{t-1}) \\ &= \overline{\lambda} + \rho_0 E_{t-1}(\varepsilon_t) + \rho_1 E_{t-1}(\varepsilon_{t-1}) \\ &= \overline{\lambda} + \rho_0 \cdot 0 + \rho_1 \varepsilon_{t-1} \\ &= \overline{\lambda} + \rho_1 \varepsilon_{t-1} \end{aligned}$$

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which we substitute into the equation for output

$$\begin{aligned} y_t &= \overline{y} + b(m_t - E_{t-1}(m_t)) \\ &= \overline{y} + b(\overline{\lambda} + \rho_0\varepsilon_t + \rho_1\varepsilon_{t-1} - (\overline{\lambda} + \rho_1\varepsilon_{t-1})) \\ &= \overline{y} + b\rho_0\varepsilon_t \end{aligned}$$

DSGE question

b) Find the level of inflation in this economy.

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We first compute the price level and its lagged value

$$\begin{aligned} p_t &= E_{t-1}(m_t) + (1-b)(m_t - E_{t-1}(m_t)) \\ &= \bar{\lambda} + \rho_1 \varepsilon_{t-1} + (1-b)(\bar{\lambda} + \rho_0 \varepsilon_t + \rho_1 \varepsilon_{t-1} - (\bar{\lambda} + \rho_1 \varepsilon_{t-1})) \\ &= \bar{\lambda} + \rho_1 \varepsilon_{t-1} + (1-b)\rho_0 \varepsilon_t \end{aligned}$$

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\begin{aligned}
p_t &= E_{t-1}(m_t) + (1-b)(m_t - E_{t-1}(m_t)) \\
&= \overline{\lambda} + \rho_1 \varepsilon_{t-1} + (1-b)(\overline{\lambda} + \rho_0 \varepsilon_t + \rho_1 \varepsilon_{t-1} - (\overline{\lambda} + \rho_1 \varepsilon_{t-1})) \\
&= \overline{\lambda} + \rho_1 \varepsilon_{t-1} + (1-b)\rho_0 \varepsilon_t \\
p_{t-1} &= \overline{\lambda} + \rho_1 \varepsilon_{t-2} + (1-b)\rho_0 \varepsilon_{t-1}
\end{aligned}
$$

which allows us to find inflation

$$
\begin{aligned}
\pi_t &= p_t - p_{t-1} \\
&= \overline{\lambda} + \rho_1 \varepsilon_{t-1} + (1-b)\rho_0 \varepsilon_t - (\overline{\lambda} + \rho_1 \varepsilon_{t-2} + (1-b)\rho_0 \varepsilon_{t-1}) \\
&= (1-b)\rho_0 \varepsilon_t + (\rho_1 - (1-b)\rho_0)\varepsilon_{t-1} - \rho_1 \varepsilon_{t-2}
\end{aligned}
$$

DSGE question

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The variance of output is given by

$$\begin{aligned} \text{Var}(y_t) &= \text{Var}(\overline{y} + b\rho_0\varepsilon_t) \\ &= \text{Var}(b\rho_0\varepsilon_t) \\ &= b^2\rho_0^2\text{Var}(\varepsilon_t) \\ &= b^2\rho_0^2\sigma_\varepsilon^2 \end{aligned}$$

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which is minimized by choosing $\rho_0^* = 0$ and allowing ρ_1^* to take any value.

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\text{Var}(\pi_t) &= \text{Var}((1-b)\rho_0\varepsilon_t + (\rho_1 - (1-b)\rho_0)\varepsilon_{t-1} - \rho_1\varepsilon_{t-2}) \\
&= \text{Var}((1-b)\rho_0\varepsilon_t) + \text{Var}((\rho_1 - (1-b)\rho_0)\varepsilon_{t-1}) + \text{Var}(\rho_1\varepsilon_{t-2})
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&= \text{Var}((1-b)\rho_0\varepsilon_t) + \text{Var}((\rho_1 - (1-b)\rho_0)\varepsilon_{t-1}) + \text{Var}(\rho_1\varepsilon_{t-2}) \\
&= (1-b)^2\rho_0^2\text{Var}(\varepsilon_t) + (\rho_1 - (1-b)\rho_0)^2\text{Var}(\varepsilon_{t-1}) + \rho_1^2\text{Var}(\varepsilon_{t-2}) \\
&= \sigma_\varepsilon^2\left[(1-b)^2\rho_0^2 + (\rho_1 - (1-b)\rho_0)^2 + \rho_1^2\right]
\end{aligned}$$

where we have used the fact that the shocks are independent over time to compute the variance of a sum as the sum of variances.

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\text{Var}(\pi_t) &= \text{Var}((1-b)\rho_0\varepsilon_t + (\rho_1 - (1-b)\rho_0)\varepsilon_{t-1} - \rho_1\varepsilon_{t-2}) \\
&= \text{Var}((1-b)\rho_0\varepsilon_t) + \text{Var}((\rho_1 - (1-b)\rho_0)\varepsilon_{t-1}) + \text{Var}(\rho_1\varepsilon_{t-2}) \\
&= (1-b)^2\rho_0^2\text{Var}(\varepsilon_t) + (\rho_1 - (1-b)\rho_0)^2\text{Var}(\varepsilon_{t-1}) + \rho_1^2\text{Var}(\varepsilon_{t-2}) \\
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where we have used the fact that the shocks are independent over time to compute the variance of a sum as the sum of variances. We see that the variance of inflation is minimized by choosing $\rho_0^* = 0$ and $\rho_1^* = 0$.

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&= \text{Var}((1-b)\rho_0\varepsilon_t) + \text{Var}((\rho_1 - (1-b)\rho_0)\varepsilon_{t-1}) + \text{Var}(\rho_1\varepsilon_{t-2}) \\
&= (1-b)^2\rho_0^2\text{Var}(\varepsilon_t) + (\rho_1 - (1-b)\rho_0)^2\text{Var}(\varepsilon_{t-1}) + \rho_1^2\text{Var}(\varepsilon_{t-2}) \\
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where we have used the fact that the shocks are independent over time to compute the variance of a sum as the sum of variances. We see that the variance of inflation is minimized by choosing $\rho_0^* = 0$ and $\rho_1^* = 0$.

e) What should ρ_0 and ρ_1 be to have both output and inflation stability?

We should choose $\rho_0^* = 0$ and $\rho_1^* = 0$ given parts c and d.

Open Economy question

Consider two open economies A and B. Assume perfect capital mobility, that the law of one price holds, and that the velocity of money is constant in both economies. The growth rates of real output are $g_{YA} = 3\%$ and $g_{YB} = 5\%$ in each country. The central bank expands money supply at $g_{MA} = 4\%$ in country A and $g_{MB} = 5\%$ in country B.

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a) Find the rate of inflation in each country.

Recall that

$$P_i Y_i = M_i v_i$$

in country $i = A, B$, where v_i is the velocity of money, and the rate of inflation is $\pi_i := \frac{\dot{P}_i}{P_i}$.

Open Economy question

a) Find the rate of inflation in each country.

Inflation is the growth rate of prices, which we compute by log-differentiating the given equation:

$$\begin{aligned} P_i Y_i &= M_i v_i \\ \ln(P_i) + \ln(Y_i) &= \ln(M_i) + \ln(v_i) \\ \frac{\dot{P}_i}{P_i} + \frac{\dot{Y}_i}{Y_i} &= \frac{\dot{M}_i}{M_i} + \frac{\dot{v}_i}{v_i} \end{aligned}$$

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\pi_i &= \frac{\dot{M}_i}{M_i} + \frac{\dot{v}_i}{v_i} - \frac{\dot{Y}_i}{Y_i}
\end{aligned}
$$

which implies that $\pi_A = 0.04 + 0 - 0.03 = 0.01$ and $\pi_B = 0.05 + 0 - 0.05 = 0$ since the velocity of money is zero.

Open Economy question

b) Find the rate of expansion of *nominal* output in each country.

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Recall that nominal output is given by $\text{nom}(Y_i) := P_i Y_i$ in country $i = A, B$. Log-differentiating gives

$$\begin{aligned} \text{nom}(Y_i) &= P_i Y_i \\ \ln(\text{nom}(Y_i)) &= \ln(P_i) + \ln(Y_i) \end{aligned}$$

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$$\begin{aligned} \text{nom}(Y_i) &= P_i Y_i \\ \ln(\text{nom}(Y_i)) &= \ln(P_i) + \ln(Y_i) \\ \frac{\dot{\text{nom}}(Y_i)}{\text{nom}(Y_i)} &= \frac{\dot{P}_i}{P_i} + \frac{\dot{Y}_i}{Y_i} \\ g_{\text{nom}(Yi)} &= \pi_i + g_{Yi} \end{aligned}$$

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g_{\text{nom}(Yi)} &= \pi_i + g_{Yi}
\end{aligned}$$

which implies that $g_{\text{nom}(YA)} = 0.01 + 0.03 = 0.04$ and $g_{\text{nom}(YB)} = 0 + 0.05 = 0.05$.

Open Economy question

c) How does the *nominal* exchange rate evolve over time?

Let $\varepsilon := \frac{\$_{US}}{\$_{FOR}}$ be the nominal exchange rate, or the units of domestic currency needed to purchase 1 unit of foreign currency. Dividing each currency by its price level gives

$$\varepsilon_R = \varepsilon \frac{P_B}{P_A} \qquad \Leftrightarrow \qquad 1 = \varepsilon \frac{P_B}{P_A} \qquad \Leftrightarrow \qquad \varepsilon = \frac{P_A}{P_B}$$

where the law of one price implies that $\varepsilon_R = 1$.

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where the law of one price implies that $\varepsilon_R = 1$. We then log-differentiate

$$\begin{aligned} \ln(\varepsilon) &= \ln(P_A) - \ln(P_B) \\ \frac{\dot{\varepsilon}}{\varepsilon} &= \frac{\dot{P}_A}{P_A} - \frac{\dot{P}_B}{P_B} \end{aligned}$$

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which implies that $g_\varepsilon = 0,01 - 0 = 0.01$.

Open Economy question

d) How does the *real* exchange rate evolve over time?

Under the assumption that the law of one price holds, the real exchange rate is equal to 1 and never changes.